A Mean-Field Theory of Cellular Automata Model for Distributed Packet Networks

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February 6, 2008

Abstract

A Mean-Field theory is presented and applied to a Cellular Automata model of distributed packet-switched networks. It is proved that, under a certain set of assumptions, the critical input traffic is inversely proportional to the free packet delay of the model. The applicability of Mean-Field theory in queue length estimation is also investigated. Results of theoretical derivations are compared with simulation samples to demonstrate the availability of the Mean-Field approach.

1 Introduction

Modelling computer networks is important for understanding the network behaviors, especially those related to the critical phenomena. Models must assign an assumption to network topology. Some of them make the assumption based on graph theory [1], while some others based on regular lattices [2, 3, 4, 5]. What kind of models is chosen depends upon the application background.

This paper focuses on architectures of distributed packet-switched networks. Some overlays, mobile *ad-hoc* networks and metropolitan area networks fall into the category of "distributed", without any central or hierarchical control and without differentiation among transit and end-user sites. On account of these characteristics, the model presented in this paper is extended from Fukś' work [3]. Actually, the same topology was studied even in the pre-Internet history, within the reports of RAND on distributed networks authored by Paul Baran and others [6, 7]. However, only survivability of the lattices was studied there. We step forward this effort into dynamic behavior of packet-switched Cellular Automata over the lattices.

On the other hand, previous works on Cellular Automata for data networks have discovered in simulations that critical traffic behavior is related to free packet delay in the networks [3]. By the help of Mean-Field Theory technique, we step forward this discovery to an approximated analytical theorem for the extended model, where the critical traffic is inversely proportional to the free delay of the network, provided the assumptions of Mean-Field Theory are available. The most basic idea in the approximation relies on simplifying the system with

an identical open Jackson network. However, the application of the Mean-Field Theory could not be exaggerated. In estimating queue length of the model, the Mean-Field approach is not accurate.

The rest parts of the paper are organized as such: section 2 describes the model and its parameters. Section 3 applies Mean-Field Theory to the model, with a certain set of approximations. Some simulation results are shown, in order to demonstrate that the Mean-Field Theory presented here are available for cases with any dimensionality and any neighborhood. Section 4 briefly discusses queue length estimated by the Mean-Field Theory, with comparison to simulation data. Finally, we summarize the works with emphasizing its significance to analysis and design of distributed network architectures.

2 Model Definitions

Cellular Automaton is a mathematical model for physical systems containing large amount of simple, identical and locally interacting units [8]. Any Cellular Automata could be defined with a 4-tuple of lattice space, neighborhood, state set, and rule of state-transition [9]. The Cellular Automata models for distributed packet-switched networks (briefly "the model" or "our model", later through the paper) are also defined as such.

2.1 Lattice

The model are defined on d-dimensional Euclidean lattice space. Originally the lattice is boundless and extended to infinity. For digital simulation, however, the lattice is often truncated in a certain d-dimensional hypercube, say L as its width. Because a distributed network has no geometrical center, the truncated lattice should be thought as periodical, i.e. the coordinate values with same remainder modulo L are identical. Therefore the lattice of the model is denoted with

$$\mathcal{L}^d \triangleq \mathbb{Z}^d \cap [0, L)^d \tag{1}$$

And the bases of the lattice space are denoted with $\mathbf{e}_i, i = 1, 2, \cdot, d$.

2.2 Neighborhood and Metric

A neighborhood is a mapping from the lattice to its power set:

$$A \quad : \quad \mathcal{L}^d \mapsto P(\mathcal{L}^d) \tag{2}$$

For the purpose of routing packets among the sites in the model, metrics are defined with the neighborhood as well. For example, von Neumann neighborhood

and the periodic Taxicab metric [1] are defined by:

$$
\begin{aligned}
A(\mathbf{x}) &= \bigcup_{i=1}^{d}\{\mathbf{x}+\mathbf{e}_i, x-\mathbf{e}_i\}, \quad \forall \mathbf{x}\in\mathcal{L}_d && (3)\\
D(\mathbf{x},\mathbf{y}) &= \sum_{i=1}^{d}\frac{L}{2}-\left||x_i-y_i|-\frac{L}{2}\right| && (4)
\end{aligned}
$$

While Moore neighborhood and the periodic Moore metric is defined as follows:

$$
\begin{aligned}
A(\mathbf{x}) &= \left(\bigcup_{y_1=-1,0,1}\cdots\bigcup_{y_d=-1,0,1}\left\{\mathbf{x}+\sum_{i=1}^{d}y_i\mathbf{e}_i\right\}\right)\setminus\{\mathbf{x}\} && (5)\\
D(\mathbf{x},\mathbf{y}) &= \max_i\left\{\frac{L}{2}-\left||x_i-y_i|-\frac{L}{2}\right|\right\} && (6)
\end{aligned}
$$

Definitions in both (4) and (6) conforms to the lattice periodicity. The two metrics are identical in 1-dimensional cases.

2.3 State and Transition

The state of a site is a first-in-first-out queue of packets, each of which contains at least the information about its destination. Each site $\mathbf{x}$'s queue length at time k, denoted as $q(\mathbf{x},k)$, is modified by both packet input and packet forwarding processes.

At each discrete moment of time k, packet enters the network from outside any site independently with an identical probability λ. The probability of more than one input packets is $o(\lambda)$. Thus, one can think there is a discrete-time Poisson source at each site. Destination of a newly entering packet is randomly selected among all possible sites with the same probability. A new packet is queued at the site where it enters the network.

At each moment, a site serves the first packet in its queue, forwarding it to one of its neighbors properly selected with the routing criteria. The service time is a constant of unity. Two criteria are applied to route selection. First, the next-hop should be selected from the neighbors, nearest to the destination in the term of given metric. The nearest neighbor set of a site $\mathbf{x}$ to destination $\mathbf{z}$ is

$$B(\mathbf{z};\mathbf{x})=\{\mathbf{y}\in A(\mathbf{x}):D(\mathbf{z},\mathbf{y})\to\min\} \tag{7}$$

Second, the next-hop should be selected from the neighbors with minimum queue length within the neighbors nearest to destination.

$$C(\mathbf{z};\mathbf{x},k)=\{\mathbf{y}\in B(\mathbf{z};\mathbf{x}):q(\mathbf{y},k)\to\min\} \tag{8}$$

Finally, if the minimum-queue nearest-to-destination neighbor set $C(\mathbf{z};\mathbf{x},k)$ contains more than one coordinates, then anyone is selected as the next-hop,

[1] Or Manhattan metric.

randomly with the same probability. If this one is current destination $\mathbf{z}$, then the packet is not queued anymore: it leaves the system.

Baran's distributed network model contains various neighborhoods [6]. To extend them into higher dimensionality, von Neumann and Moore neighborhoods are definitely well defined while other neighborhoods might not. Fukś' two-dimensional von Neumann Cellular Automata model allows non-periodical property [3], but it's less close to the case of distributed networks where all the sites are identical in topology.

3 Mean-Field Theory

Any queueing system has a critical traffic as the upper bound of the input traffic, so that the system converges into a stable state instead of going far from stability. This critical traffic is just the service rate μ in a single queueing-service system, while it is different for networks.

Ohira has shown that the critical traffic is related to the free delay of the network [5] and Fukś has uncovered that, in two-dimensional von Neumann Cellular Automata model, the sufficient and necessary condition for stability of the model is $\lambda < 1/\bar{\tau}_0$, or equivalently,

$$\lambda_c = \frac{1}{\bar{\tau}_0} \tag{9}$$

where $\bar{\tau}_0$ is the average delay of a free packet without being queued anywhere. These previous results are observed in simulations. Now we prove that the law of (9) is an analytical result under certain assumptions and is available for whatever dimensionality and neighborhoods rather than only for two-dimensional von Neumann cases.

3.1 Approximation to Open Jackson Network

The model defined in the previous section has the property of open queueing networks, i.e. all packets enter the system from outside and finally leave at the destination. This inspires utilizing ever-known conclusions in open Jackson network.

Open Jackson network is of Markovian queueing network, i.e. packet arrival is a Poisson process and the service time of each site conforms to exponential distribution. The Jackson theorem presents the condition of stability as well as the queue length distribution in stable state [10, 11]. In the Jackson theorem, a parameter $\sigma(\mathbf{x})$, called as "site traffic", is defined as the traffic observed at site $\mathbf{x}$ in the system and we have the traffic equilibrium equations

$$\sigma(\mathbf{x}) = \lambda(\mathbf{x}) + \sum_{\mathbf{y}} \sigma(\mathbf{y}) r_{\mathbf{yx}}, \quad \forall \mathbf{x} \tag{10}$$

where $r_{\mathbf{yx}}$ is the forwarding probability from site $\mathbf{y}$ to $\mathbf{x}$ and

$$\sum_{\mathbf{x}} r_{\mathbf{yx}} = 1 - r_{\mathbf{y},\infty}, \quad \forall \mathbf{y} \tag{11}$$

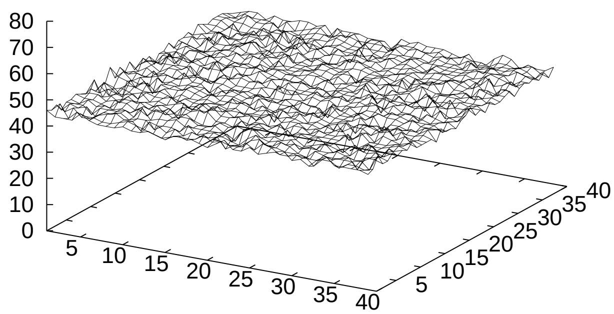


Figure 1: Queue length field of a sample with $d = 2, L = 40, \lambda = 0.08, k = 1000$, von Neumann neighborhood, and periodic taxicab metric. Queue lengths are so close to each other among all the sites that assuming they have an identical σ is reasonable.

where $r_{\mathbf{y},\infty}$ represents the probability of leaving.

Our model, however, contains sites over a lattice space, each of which is a discrete-time M/D/1 queueing system. Unfortunately, to the best knowledge of the authors, there are hardly few approaches of queueing networks consisting of M/D/1 systems. Then the constant service time is replaced by a exponential distribution with the mean value of $1/\mu = 1$. Furthermore, we ignore the details in route selection and approximate it with a simple, time-invariant, and destination-free probability, $r_{\mathbf{yx}}$,

Thus, the model is approximated with a continuous-time open Jackson network over the lattice space, where input traffic is an identical Poisson process at each site with parameter $\lambda(\mathbf{x}) = \lambda$, and each site serves the traffic with parameter $\mu = 1$.

3.2 Mean-Field Theory for Critical Traffic

Observing samples of queue growth processes in the model (Figure 1), one can easily summarized that site traffic $\sigma(\mathbf{x})$ seems to be a constant without difference referring to the coordinates. This implies that a Mean-Field Theory can be developed in order to get the critical traffic law of the open Jackson network derived from the model.

Mean-Field Theory is an approximate technique widely used in statistical physics, which treats the order-parameter as spatially constant [12]. For our model, the Mean-Field approximation aims at an identical parameter σ such

that

$$\sigma(\mathbf{x}) = \sigma, \quad \forall \mathbf{x} \in \mathcal{L}^d \tag{12}$$

Three heuristic conditions, called as Mean-Field Theory assumptions, support this approximation.

1. Isotropy Lattice space, either infinite or periodic, is isotropic in geometry. Therefore, it is reasonable to assume that, at any moment, any site forwards the first packet with a same probability to its neighbors. That is

$$\begin{aligned} r_{\mathbf{yx}} &= r_{\mathbf{y}} \\ &= \frac{1}{|A(\mathbf{y})|}\left(1 - r_{\mathbf{y},\infty}\right), \quad \forall \mathbf{x} \in A(\mathbf{y}), \quad \forall \mathbf{y} \in \mathcal{L}^d \end{aligned} \tag{13}$$

2. Homogeneity Further, it is assumed that the forwarding probability are not different among all the sites. In addition to the formula (13) and the fact $|A(\mathbf{y})| \equiv A, \quad \forall \mathbf{y} \in \mathcal{L}^d$, we have

$$\begin{aligned} r_{\mathbf{y}} &= r = \frac{1}{A}\left(1 - r_\infty\right) \\ r_{\mathbf{y},\infty} &= r_\infty, \quad \forall \mathbf{y} \in \mathcal{L}^d \end{aligned} \tag{14}$$

3. Spatial and temporal ergodicity It is assumed that the queue length process $q(\mathbf{x}, k)$ is spatially ergodic, i.e.

$$E\{q(\mathbf{x}, k)\} = \lim_{L\to\infty} \frac{1}{|\mathcal{L}^d|} \sum_{\mathbf{x}\in\mathcal{L}^d} q(\mathbf{x}, k), a.s. \tag{15}$$

And furthermore, it is also temporally ergodic as long as the stable state $q(\mathbf{x}) \triangleq \lim_{k\to\infty} q(\mathbf{x}, k)$ is achieved. Later on, we denote $\bar{q}$ for either the ensemble, or temporal, or spatial average (in the stable state if it exists) of queue length processes on sites over the lattice.

From (13) and (14), note that $\lambda(\mathbf{x})$ are identical to λ, the traffic equations (10) is simplified to

$$\sigma(\mathbf{x}) = \lambda + \left[\frac{1}{A}(1 - r_\infty)\right] \sum_{\mathbf{y}} \sigma(\mathbf{y}), \quad \forall \mathbf{x} \in \mathcal{L}^d \tag{16}$$

For linear equations (16) are symmetric to permutations of $\{\sigma(\mathbf{x})\}$, and the number of equations are equal to the number of unknown variables, it is definite that they have a unique solution set which is satisfying (12).

Then equation (16) are reduced to a single equation referring to σ:

$$\sigma = \lambda + (1 - r_\infty)\sigma \tag{17}$$

and finally it is solved that

$$\sigma = \frac{\lambda}{r_\infty} \tag{18}$$

The value of r_∞ has not yet determined. We'd like to present the Mean-Field version of Jackson Theorem first and then derive r_∞ by applying the well-known Little's Law to the mean value of packet lifetime.

With help of the assumptions above, and the Jackson Theorem, we have

Theorem 1 *Under the assumptions of Mean-Field Thoery, the queueing network of the model converges to stable state if and only if*

$$\rho \triangleq \frac{\sigma}{\mu} = \frac{\lambda}{r_\infty} < 1 \tag{19}$$

And in the stable state, queue length on each site, $q(\mathbf{x})$, conforms to an identical geometric distribution:

$$P(q(\mathbf{x}) = n) = (1-\rho)\rho^n, \quad \forall n \geq 0, \quad \forall \mathbf{x} \in \mathcal{L}^d \tag{20}$$

And the mathematical expectation of stable queue length is:

$$\bar{q} = \frac{\rho}{1-\rho}, \quad \forall \mathbf{x} \in \mathcal{L}^d \tag{21}$$

Now from the Little's Law of arbitrary queueing system, it holds in the stable state that

$$\bar{q} = \lambda \bar{\tau} \tag{22}$$

where $\bar{\tau}$ is the mathematical expectation of packet lifetime in stable state. When all the sites have identical queue length, lifetime of a packet is independent upon the path that it pass through, and is equal to the free delay $\bar{\tau}_0$ plus the total time of being queued. Note that the packet must be queued $\bar{\tau}_0$ times, and apply the third assumption to regard queue length of each site on a packet path is constant and equal to the average $\bar{q}$, then we have

$$\bar{\tau} = \bar{\tau}_0 + \bar{\tau}_0 \bar{q} \tag{23}$$

Apply (23) to (22), we obtain an equation

$$\begin{aligned} \bar{q} &= \lambda(\bar{\tau}_0 + \bar{\tau}_0 \bar{q}) \\ \text{and} \quad \bar{q} &= \frac{\lambda \bar{\tau}_0}{1 - \lambda \bar{\tau}_0} \end{aligned} \tag{24}$$

Recall the formulae (19), (21) in the Theorem 1 and compare them to the equation (24), we have the following corollary, which represents the law of critical traffic as a function of the free delay in the model[2].

[2]The third assumption plays an important role here. Note that in formula (21), $\bar{q}$ is, in fact, the ensemble average; while in Little's Law (24), $\bar{q}$ is actually a time average. Without the ergodicity assumption, the two formula could not be combined.

Corollary 2 *In the Mean-Field Theory of the model, the packet's leaving probability and the site traffic are respectively*

$$r_\infty = \frac{1}{\bar{\tau}_0} \tag{25}$$

$$\text{and} \quad \sigma = \lambda \bar{\tau}_0 \tag{26}$$

And the stability condition is equivalent to

$$\lambda < \frac{1}{\bar{\tau}_0} \tag{27}$$

or equally to say the critical input traffic is $\lambda_c = \frac{1}{\bar{\tau}_0}$.

The corollary, esp. the formula (26), shows that the free delay of a network does significantly impact on the critical traffic. Actually, delay *amplifyings* input traffic to site traffic linearly; or equivalently to say, free delay in a network decrease the service capability of the sites.

It is emphasized that, in the Mean-Field Theory demonstrated above, there is not any requirement to lattice dimensionality, nor its neighborhood type. The analytical result of Theorem 1 and its Corollary is universally available, provided the Mean-Field Theory assumptions are conforming to physical properties of the model.

3.3 Simulation Samples

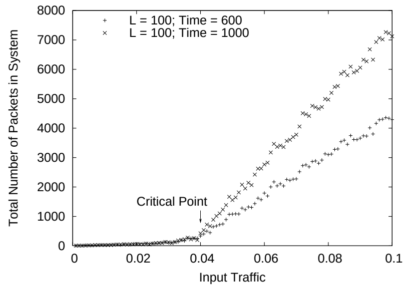
As the upper bound of input traffic for existence of stable state, the critical behavior in an instance of the model may be demonstrated by its queue length processes or the total number of packets in the entire system, say $Q(k) \triangleq \sum_{\mathbf{x}} q(\mathbf{x}, k)$. When the running time k is large enough, the model is thought of being stable if the total number $Q(k)$ is almost unchanged as k goes larger. If the input traffic exceeds the critical point, then $Q(k)$ will increase steadily to infinity. These phenomena are shown in Figure 2, where several sample cases with variety of parameters or topology characteristics are provided. Figure 2(c) is similar to the case that [3] has provided. However, we have extended the results on critical traffic to far more general environments.

4 Queue Length Estimation

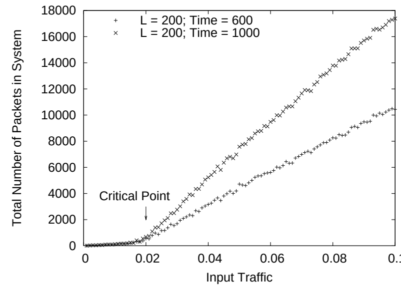
As any other Mean-Field approaches for statistical-physical systems, the Mean-Field Theory presented here can *not* be treated as an accurate quantitative result, though it almost accurately predicts the critical behavior of the model.

4.1 Queue Length of Stable State

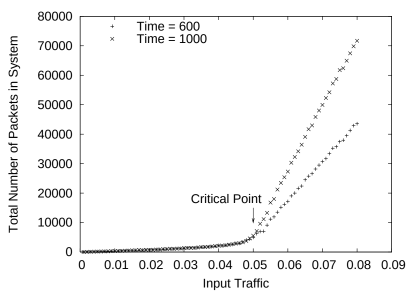
Theorem 1 has given the mean value for the queue length on any site in the stable state, if it exists. However, the result overestimates it a little.



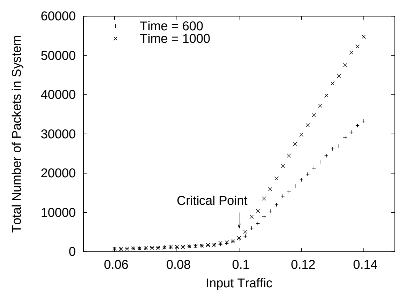
(a) $d = 1$, $L = 100$, $\bar{\tau}_0 = 25$, $\lambda_c = 0.04$



(b) $d = 1$, $L = 200$, $\bar{\tau}_0 = 50$, $\lambda_c = 0.02$



(c) von Neumann lattice, $d = 2$, $L = 40$, $\bar{\tau}_0 = 20$, $\lambda_c = 0.05$



(d) Moore lattice, $d = 2$, $L = 30$, $\bar{\tau}_0 = \frac{1801}{180} \simeq 10.0$, $\lambda_c = 0.10$

Figure 2: Simulation samples to show the critical behavior. With whatever dimensionality and whatever neighborhood, the model's critical traffic λ_c is almost equal to $1/\bar{\tau}_0$.

The overestimation may originated from approximating the model to an open Jackson network. Each site in the model is defined as an M/D/1 queueing system, while it becomes to M/M/1 in the Jackson network. It has been proved that, for Markovian routing schemes, an M/D/1 queueing network has less average queueing delay (or, equivalently, less queue length) in stable state than its M/M/1 counterpart [13].

4.2 Fluid approximation of the Mean-Field Theory

On the other hand, when the value of input traffic exceeds the critical point, it is presented in the simulations that queue length of a site approximately grows as a linear function to both time and input traffic. To get this phenomenon explained, we combine the assumptions of the Mean-Field Theory and (25) with the Fluid approximation in queueing theory [14].

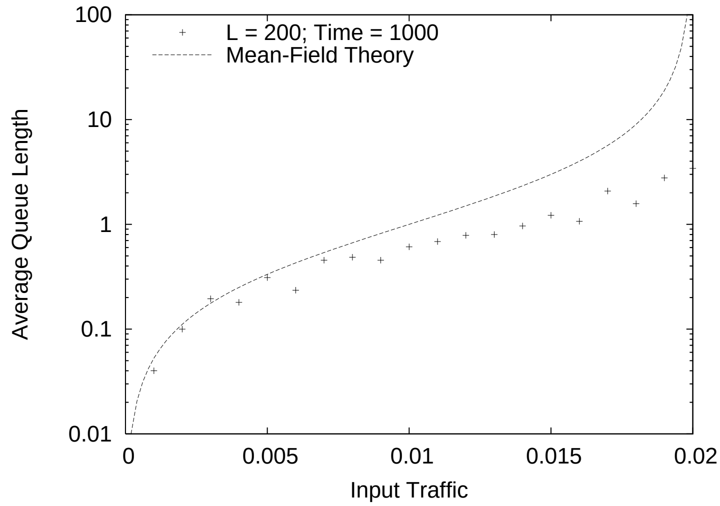


Figure 3: Comparison simulation to Mean-Field Theory on a one-dimensional sample with $L = 200, k = 1000$, for stable state queue length. The Mean-Field Fluid approximation overestimates the average queue length.

The Fluid approximation, based on the law of large numbers, replaces discontinuous stochastic arrival and departure processes with continuous deterministic versions. Let $\overline{\alpha(\mathbf{x}, t)}$ and $\overline{\delta(\mathbf{x}, t)}$ represents, respectively, the two continuous processes for packet arrival and departure happening at site $\mathbf{x} \in \mathcal{L}^d$ in our model. Then for the queue length approximation $\overline{q(\mathbf{x}, t)}$, we have

$$\overline{q(\mathbf{x}, t)} = \overline{\alpha(\mathbf{x}, t)} - \overline{\delta(\mathbf{x}, t)}, \quad \forall \mathbf{x} \in \mathcal{L}^d \tag{28}$$

Arrivals are resulted by both traffic input and forwarding events, and the departure rate is constant $\mu = 1$. Therefore, we have the Fluid version of (10).

$$\overline{\alpha(\mathbf{x}, t)} = \overline{\alpha(\mathbf{x}, 0)} + \int_0^t \lambda dy + \sum_{\mathbf{y} \in A(\mathbf{x})} r_{\mathbf{yx}} \overline{\delta(\mathbf{y}, t)}, \tag{29}$$

$$\overline{\delta(\mathbf{x}, t)} = \overline{\delta(\mathbf{x}, 0)} + \int_0^t dy = \overline{\delta(\mathbf{x}, 0)} + t, \quad \forall \mathbf{x} \in \mathcal{L}^d \tag{30}$$

With (25), i.e. $r_{\mathbf{yx}} = (1 - 1/\bar{\tau}_0)/A$, another corollary is obtained.

Corollary 3 *In the Mean-Field Theory of the model, approximated with Fluid model, queue length at any site is growing linearly if the input traffic exceeds the critical value, and the growth rate is*

$$\frac{d\overline{q}}{dt} = \lambda - \frac{1}{\bar{\tau}_0} = \lambda - \lambda_c, \quad \forall \mathbf{x} \in \mathcal{L}, t \to \infty \tag{31}$$

The result is compared to simulation in a way with either time fixed and input traffic variant or vise-versa (Figure 4). The comparison focuses on the growth rate. It is reasonable that the Mean-Field Fluid approximation a little underestimates the rate of queue growth because Fluid approaches replace the stochastic processes with deterministic (D/D/1) systems. The larger the input traffic is, the more the rate of queue growth estimated with Mean-Field Fluid approach is close to the simulation.

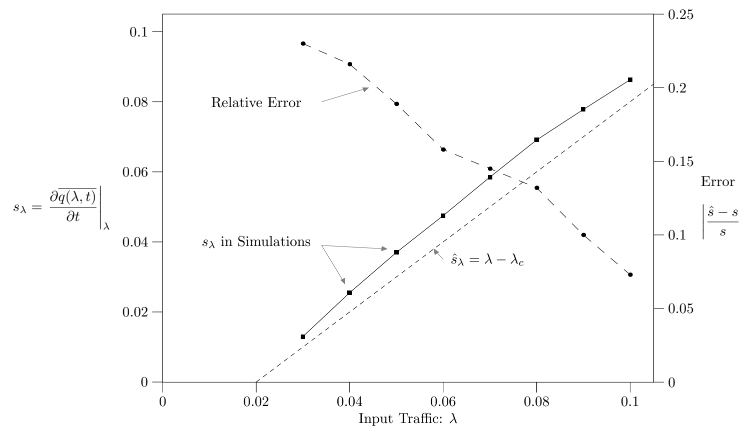
5 Summary

The work is a part of research efforts on distributed network architectures. The property of being distributed makes a network much more reliable but a little less efficient than a hierarchical decentralized architecture, such as today's Internet. However, in some circumstances, the network is so dynamically organized (such as *ad hoc* network) that reliability is far important rather than the efficiency, even it is not possible to build any centralized control or any hierarchy. In some other circumstances such as metropolitan-area-networks, because the speed of physical links has greatly increased than the age of early Internet, distributed architecture may perform better in both reliability and efficiency.

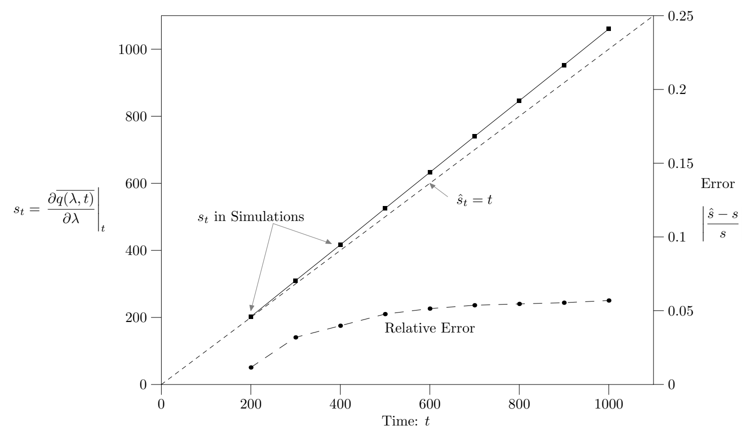
This paper develops a modelling method for studying distributed network behaviors. The introduced Cellular Automata model can be applied not only to analyzing but also to designing architectures, where a topology like regular lattices is able to be implemented. It promotes the previous works of simulation to an approximated analytical result, where the law of critical traffic is proved under a certain set of Mean-Field assumptions. Being advantageous to finite simulations, the analytical result refines the conditions of the law of critical traffic. Under these conditions, i.e. isotropy, homogeneity and ergodicity, the critical traffic is determined by the free delay of the network. Therefore, one may improve the critical behavior of a network by improving the free delay of its topology equivalently.

On the other hand, the Mean-Field Theory is not able to accurately estimate some quantities such as average queue length in the model. Generally speaking, the farther the input traffic is away from the the critical point, the better the Mean-Field theory estimates queue length behavior, no matter what kind of state the model is in, stable or far from stability. In practice, quality control protocol may predict the queueing tendency under either definitely heavy or definitely light input and do actions in response.

It is true that, once the assumptions are far from reality, the Mean-Field Theory will be broken. For example, when some sites in the model cease working, packets must bypass these sites and therefore the isotropy of the model is destroyed. Since the critical behavior depends upon the maximum site traffic, it could be easily derived from (10) and has been demonstrated in simulations that in such a case the critical input traffic is significantly less than that of the Mean-Field Theory, and improving free delay is still important because it upper-bounds the maximum-possible critical traffic.



(a) With fixed input traffic, the Mean-Field Fluid approach estimates that the slope of queue length to time is $\lambda - \lambda_c$, which is near to but a little less than the result of simulations.



(b) With fixed time, the Mean-Field Fluid approach estimates that the slope of queue length to input traffic is t, which is near to but a little less than the simulation results too.

Figure 4: Comparison the simulation results to Mean-Field Fluid approximation on one-dimensional samples with $L = 200$, for queue length growth far from stability. The Mean-Field Fluid approximation is near to but a little underestimates the queue growth rate.

Quantitatively analyzing non-isotropic, non-homogeneous and non-ergodic models, which are more close to real systems, are of the successive work after this paper. Like studying any new statistical-physical systems, one should get deep into the local details of reaction, where the fluctuations are ignored by the Mean-Field Theory.

References

[1] Tretyakov, A., Takayasu, H., Takayasu, M.: Phase transition in a computer network model. Physica A **253** (1998) 315–322

[2] Deane, J., Smythe, C., Jefferies, D.: Self-similarity in a deterministic model of data transfer. Journal of Electronics **80** (1996) 677–691

[3] Fukś, H., Lawniczak, A.T.: Performance of data networks with random links. Mathematics and Computers in Simulation **51** (1999) 101–117

[4] Liu, F., Ren, Y., Shan, X.M.: A simple cellular automata model for packet transport in internet. Acta Physica Sinica **51** (2002) 1175–1180

[5] Ohira, T., Sawatari, R.: Phase transition in a computer network traffic model. Physics Review E **58** (1998) 193–195

[6] Baran, P.: History, alternative approaches and comparisons. In: On Distributed Communications. Volume V. RAND RM-3097-PR (1964)

[7] Baran, P.: Introduction to distributed communications network. In: On Distributed Communications. Volume I. RAND RM-3420-PR (1964)

[8] Wolfram, S.: Universality and complexity in cellular automata. Physica D **10** (1984) 1–35

[9] Weimar, J.R.: Cellular Automata for Reactive Systems. Phd dissertation, Université Libre de Bruxelles, Belgium (1995)

[10] Kleinrock, L.: Queueing Systems. Volume I: Theory. Wiley Interscience (1976)

[11] Sheng, Y.Z.: Queueing Theory and its Applications to Computer Communication (in Chinese). Beijing University of Post-Telecommunication Press (1998)

[12] Chaikin, P.M., Lubensky, T.C.: Principles of Condensed Matter Physics. Cambridge University Press (1997)

[13] Harchol-Balter, M.: Network Analysis without Exponential Assumptions. Phd dissertation, University of California at Berkeley (1996)

[14] Kleinrock, L.: Queueing Systems. Volume II: Computer Applications. Jon Wiley & Sons (1976)